We may thus write that

$$(\partial H/\partial p)_T = f'(p) = (\partial H/\partial p)_S + (\partial H/\partial S)_p(\partial S/\partial p)_T$$

$$= v - T\,(\partial v/\partial T)_p.$$

Dividing by T^2 we find that

$$f'(p)/T^2 = -(\partial(v/T)/\partial T)_p\,.$$

When integrated this yields the implied form for the equation of state:

$$v = f'(p) + Tg(p) \tag{2.2}$$

where $g(p) = (\partial v/\partial T)_p$

In a similar way it can be shown that if specific heat at constant volume, C_v, is a function of T alone, $C_v = C_v\,(T)$, then it must follow that p, T, and v are related by a Grüneisen equation:

$$p = f(v) + Th(v) \tag{2.3}$$

Both Eqs. (2.2) and (2.3) are remarkably simple. The (p, v, T) surfaces that can be represented by either of these can be constructed from a "bamboo-place-mat." In the first case the straight sticks lie in planes of constant p; in the second they lie in planes of constant v.

Eq. (2.3) has the same form as the Mie-Grüneisen equation:

$$p = p_k(v) + \Gamma C_v\,(T-T_o)/v \tag{2.4}$$

where p_k is pressure on the isotherm $T = T_o$. With the assumption that $C_v = C_v(T)$, Eqs. (2.2) and (2.3) then imply that

$$(\partial(\Gamma C_v)/\partial T)_v = 0.$$

This condition implies that $(\partial\Gamma/\partial T)_v \neq 0$ unless C_v = const. While it is very likely true that Γ does depend upon T (4), the only available theories of any generality suppose that $\Gamma = \Gamma(v)$ (1). Fowles has obtained a more general compatibility relation for Γ and C_v (5):

$$(\partial C_v/\partial \ln v)_T = (\partial(\Gamma C_v)/\partial \ln T)_v$$

It turns out that this is satisfied by the Debye theory of specific heats. However, any dependence of Debye temperature on temperature violates the condition.

Despite these difficulties, Eq. (2.3) has been commonly used to extend pressure-volume data determined from shock studies into off-Hugoniot regions (1). Doran (6) has gone even farther to show that quite reasonable representations of the equations of state of solids can be obtained with Γ/v = constant.

In view of the limitations on Γ and C_v, it is unlikely that the zero degree isotherms calculated using Eq. (2.4) and the Slater or Dugdale-McDonald relation for Γ are physically reliable. Their principal virtue is that they provide a consistent and reproducible procedure for determining a reference curve. An alternative procedure is to calculate the isotherm passing through the initial state of the Hugoniot. This avoids some of the difficulties associated with extrapolating to 0°K. It introduces some new ones inasmuch as there is now no theory for